P. P. SAVANI UNIVERSITY

First Semester of B.Sc. Examination
February-2022
SSCH1050-Mathematics

10.02.2022, Thursday   **Time: 12:00 p.m. to 2:30 p.m.**   **Maximum Marks: 60**

Instructions:
1. The question paper comprises two sections.
2. Section I and II must be attempted in separate answer sheets.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculators is allowed.

Section-I (Total Marks - 30)

Q.1   **Short Questions**   [10]

1.1   **Objectives**   [05]

1.1a   The value of i^i is:

A. 0

B. $e^{-\pi}$

C. $2\ e^{-\pi/2}$

D. $e^{-\pi/2}$

1.1b   If $f(x) = 2^x$ then the range of the function is

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(-\infty, \infty)$

D. None

1.1c   The modulus of (1- i) is

A $\sqrt{2}$

B 0

C 1

D *none*

1.1d   The value of cos 5π *is*

A 0

B 1

C -1

D None

1.1e   The range of the function $f(x) = sinx$ *is*

A. $(-\infty, \infty)$
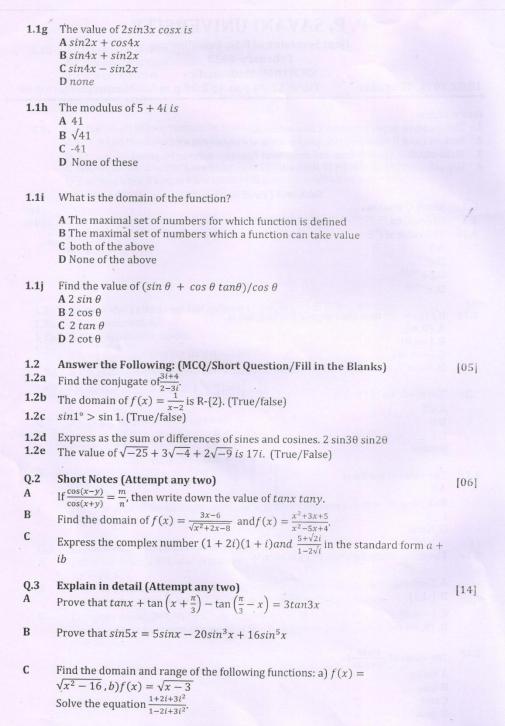
B. $[-1,1]$

C. $(-\infty, 0)$

D. $(0, \infty)$

1.1f   The value of $\frac{sin\ 2x}{1+cos2x}$ *is*

A tan2x

B tanx

C sinx

D cosx

1.1g The value of $2\sin3x \cos x$ is

A $\sin2x + \cos4x$

B $\sin4x + \sin2x$

C $\sin4x - \sin2x$

D *none*

1.1h The modulus of $5 + 4i$ is

A 41

B $\sqrt{41}$

C -41

D None of these

1.1i What is the domain of the function?

A The maximal set of numbers for which function is defined

B The maximal set of numbers which a function can take value

C both of the above

D None of the above

1.1j Find the value of $(\sin\theta + \cos\theta\tan\theta)/\cos\theta$

A $2\sin\theta$

B $2\cos\theta$

C $2\tan\theta$

D $2\cot\theta$

1.2 **Answer the Following: (MCQ/Short Question/Fill in the Blanks)** [05]

1.2a Find the conjugate of $\frac{3i+4}{2-3i}$.

1.2b The domain of $f(x) = \frac{1}{x-2}$ is R-{2}. (True/false)

1.2c $\sin1° > \sin 1$. (True/false)

1.2d Express as the sum or differences of sines and cosines. $2\sin3\theta\sin2\theta$

1.2e The value of $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$ is $17i$. (True/False)

Q.2 **Short Notes (Attempt any two)** [06]

A If $\frac{\cos(x-y)}{\cos(x+y)} = \frac{m}{n}$, then write down the value of $\tan x \tan y$.

B Find the domain of $f(x) = \frac{3x-6}{\sqrt{x^2+2x-8}}$ and $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$.

C Express the complex number $(1 + 2i)(1 + i)$ and $\frac{5+\sqrt{2}i}{1-2\sqrt{i}}$ in the standard form $a + ib$

Q.3 **Explain in detail (Attempt any two)** [14]

A Prove that $\tan x + \tan\left(x + \frac{\pi}{3}\right) - \tan\left(\frac{\pi}{3} - x\right) = 3\tan3x$

B Prove that $\sin5x = 5\sin x - 20\sin^3x + 16\sin^5x$

C Find the domain and range of the following functions: a) $f(x) = \sqrt{x^2 - 16}$, b) $f(x) = \sqrt{x - 3}$

Solve the equation $\frac{1+2i+3i^2}{1-2i+3i^2}$.

Q.1   **Short Questions**   [10]

1.1   **Objectives**   [05]

1.1a   If $A = \begin{bmatrix} cosx & -sinx \\ sinx & cosx \end{bmatrix}$ and $A + A' = I$, then the value of x is

    A $\frac{\pi}{6}$

    B $\frac{\pi}{3}$

    C π

    D $\frac{3\pi}{2}$

1.1b   If A, B are symmetric matrices of same order, then AB – BA is a

    A skew symmetric matrix

    B symmetric matrix

    C zero matrix

    D Identity matrix

1.1c   If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

    A $1 + \alpha^2 + \beta\gamma = 0$

    B $1 - \alpha^2 + \beta\gamma = 0$

    C $1 - \alpha^2 - \beta\gamma = 0$

    D $1 + \alpha^2 - \beta\gamma = 0$

1.1d   Which one of them is correct? $\vec{A} \times \vec{B} =$

    A AB $sin\theta\hat{n}$

    B AB $sin\theta\vec{n}$

    C AB $cos\theta\hat{n}$

    D *none*

1.1e   The angle between vectors $\vec{A}$ and $\vec{B}$ is given by

    A $cos\theta = \frac{\vec{A}.\vec{B}}{2|A||B|}$

    B $tan\theta = \frac{\vec{A}.\vec{B}}{|A||B|}$

    C $sin\theta = \frac{\vec{A}.\vec{B}}{|A||B|}$

    D $cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|}$

1.1f   The point on y-axis which is at a distance of $\sqrt{10}$ units from the point $(1,2,3)$.

    A $(0,2,0)$

    B $(0,1,0)$

    C $(2,1,0)$

    D $(3,1,0)$

1.1g   Find the determinant $\begin{vmatrix} cos40° & -cos40° \\ sin50° & cos50° \end{vmatrix}$

    A 0

    B 1

    C -1

    D 2

1.1h If the matrix A is both symmetric and skew symmetric, then
A A is a diagonal matrix
B A is a zero matrix
C A is a square matrix
D None of these

1.1i Can the result of two vectors be zero?
A Yes, when the 2 vectors are same in magnitude and direction
B No
C Yes, when the 2 vectors are same in magnitude but opposite in sense
D Yes, when the 2 vectors are same in magnitude making an angle of 2/3 with each other

1.1j If the sum of two unit vectors is a unit vector, then magnitude of difference is
A $\sqrt{2}$
B $\sqrt{3}$
C $\frac{1}{\sqrt{2}}$
D $\sqrt{5}$

1.2 **Answer the Following: (MCQ/Short Question/Fill in the Blanks)** [05]
1.2a Define unit vector.
1.2b Define: symmetric matrix.
1.2c If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, calculate its determinant.
1.2d What is the difference between scalar and vector?
1.2e Find value of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

Q.2 **Short Notes (Attempt any two)** [06]
A Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

B Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

C Find the area of parallelogram whose adjacent sides are $i - 2j + 3k$ and $2i + j - 4k$.

Q.3 **Explain in detail (Attempt any two)** [14]
A Solve the system of linear equations: $x + 2y - z = 1, 2x + 3y + z = 2$ and $x + 3y - 2z = 1$
B Test the consistency of the following equations and solve them if possible: $x - y + 2z = 2, 2x + y + 4z = 7, 4x - y + z = 4$ using determinants.
C Compute the angle between two vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$.
Find the angle between two vectors $5i - j + k$ and $i + j - k$.